

# Arbitrarily Oriented Microstrip Lines Coupled Through an Inclined Slot in the Common Ground Plane

Y. M. M. Antar, Z. Fan, and A. Ittipiboon

**Abstract**— An analysis of arbitrarily oriented microstrip lines coupled through an inclined slot in the common ground plane is presented. The method of analysis is based on the spectral domain approach and reciprocity theorem. Computed scattering parameters are compared with other available measured and computed data for the case of parallel microstrip lines, showing good agreements. It is found that for arbitrarily oriented microstrip lines, by varying slot inclination angle, coupling level can be widely controlled and there exists maximum coupling for a certain value of slot inclination angle. Effects of slot length on coupling and losses are also investigated.

## I. INTRODUCTION

THERE has been considerable interest in recent years in the development of multilevel monolithic microwave integrated circuits and multilayer phased antenna arrays [1]. For these multilayer structures, surface-to-surface transitions between printed transmission lines are required for the transfer of power between two adjacent layers. Among these transitions, slot-coupled microstrip lines play an important role for the design of multilayer feed networks, couplers, and filters. Several papers have recently been published dealing with theoretical and experimental investigations of these slot-coupled microstrip lines [2]–[6]. Most analyses reported so far were based on the assumption that coupled microstrip lines are parallel and the coupling slot is perpendicular to the lines, and to date no results have been reported for arbitrarily oriented microstrip lines. Some results for an inclined slot, but between parallel microstrip lines, have been reported recently in [6]. Also, in [6] an enclosed structure was considered to avoid dealing with poles, so effects of radiation and surface waves were not accounted for. However, in practical circuit and antenna designs, coupled lines may be arbitrarily oriented and the properly inclined slot may also be required to control the coupling between the lines. The purpose of this letter is to present an analysis of the general case of the arbitrarily oriented slot-coupled microstrip lines where the slot is inclined to two lines at two different angles (see Fig. 1) and to investigate the effects of slot inclination angle on scattering parameters. The spectral domain approach is used to find

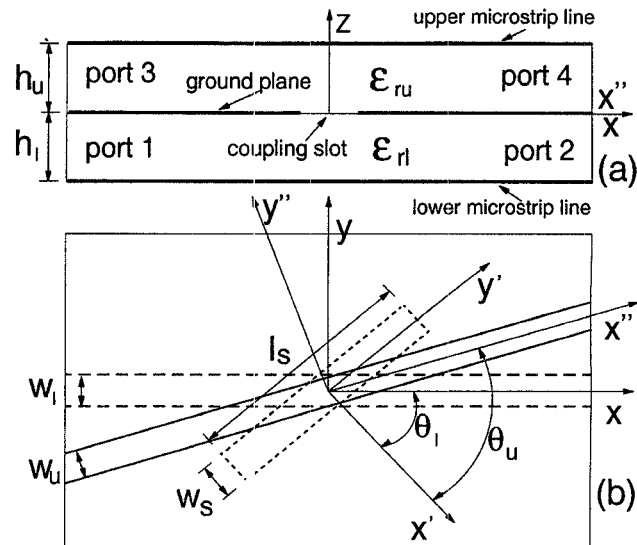


Fig. 1. (a) Side view and (b) top view of slot-coupled microstrip lines, where the slot is inclined to both lines at different angles.

propagation characteristics of microstrip lines and the fields generated by the slot. The reciprocity theorem is applied to derive the expressions for scattering parameters of this four-port structure in terms of the electric field in the slot. This method, combining the spectral domain approach and the reciprocity theorem, is numerically efficient and should also find useful applications to other related slot-coupling problems.

## II. METHOD OF ANALYSIS

The ports of the structure under investigation are defined in Fig. 1. To compute the scattering parameters  $[S_{ij}]$  of this four-port network, the quasi-TEM mode phase constants ( $\beta_l, \beta_u$ ), characteristic impedances, and field eigenvectors of infinitely long lower ( $z < 0$ ) and upper ( $z > 0$ ) microstrip lines have to be first found. This is done by employing Galerkin's procedure in the spectral domain, where the electric currents on the lines are expanded in terms of Chebyshev polynomials weighted by an appropriate singular function.

The incident quasi-TEM mode field is first launched from  $x = -\infty$  into port 1. When the reciprocity theorem [2] is applied to the total fields including the fields generated by the slot and the fields of the positive travelling wave of the quasi-

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TEM mode of the lower microstrip line,  $S_{11}$  can be obtained as follows:

$$S_{11} = \frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{\sin F_1^l}{F_1^l} \cdot \tilde{E}_{x'}^s(-\kappa_y \cos \theta_l - \beta_l \sin \theta_l) F_2^l d\kappa_y. \quad (1)$$

Similarly, applying the reciprocity theorem to the total fields and the fields of the negative travelling-wave results in the following expression for  $S_{21}$

$$S_{21} = 1 - \frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{\sin F_1^l}{F_1^l} \cdot \tilde{E}_{x'}^s(\kappa_y \cos \theta_l + \beta_l \sin \theta_l) F_2^l d\kappa_y \quad (2)$$

where

$$\tilde{E}_{x'}^s(\alpha) = \int_{-0.5l_s}^{0.5l_s} E_{x'}^s(y') e^{j\alpha y'} dy' \quad (3)$$

$$F_1^l = 0.5w_s(-\kappa_y \sin \theta_l + \beta_l \cos \theta_l) \quad (4)$$

$$F_2^l = \cos \theta_l \tilde{h}_y^l(\kappa_y, 0) + \sin \theta_l \tilde{h}_x^l(\kappa_y, 0). \quad (5)$$

$\tilde{h}_x^l(\kappa_y, 0)$  and  $\tilde{h}_y^l(\kappa_y, 0)$  are Fourier transforms of  $x$  and  $y$  magnetic field components of the quasi-TEM mode of the lower microstrip line. Only the  $x'$  component of the electric field  $E_{x'}$  in the slot has been considered, since the slot width  $w_s$  is assumed here to be electrically short. Similarly, by applying the reciprocity theorem to the total fields and the fields of the quasi-TEM mode of the upper microstrip line, the scattering parameters  $S_{31}$  and  $S_{41}$  can also be obtained

$$S_{31} = -\frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{\sin F_1^u}{F_1^u} \cdot \tilde{E}_{x'}^s(-\kappa_y \cos \theta_u - \beta_u \sin \theta_u) F_2^u d\kappa_y \quad (6)$$

$$S_{41} = \frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{\sin F_1^u}{F_1^u} \cdot \tilde{E}_{x'}^s(\kappa_y \cos \theta_u + \beta_u \sin \theta_u) F_2^u d\kappa_y \quad (7)$$

where  $F_1^u$  and  $F_2^u$  can be obtained from the right-hand side of (4) and (5) by replacing  $l$  by  $u$ , respectively. An integral equation can be obtained by enforcing the continuity condition for the magnetic field through the slot, and it can be solved for the slot electric field expansion coefficients by applying the method of moments.

### III. RESULTS

To assess the accuracy of this analysis, our computed results for the case of parallel lines are compared with measured and computed data available in the literature. The first comparison is with measured and computed data reported by Herscovici and Pozar in [2] for parallel microstrip lines with a perpendicular slot. The comparison is shown in Fig. 2 for  $|S_{11}|$  and  $|S_{21}|$ , and agreement between these data is found to be good. In Fig. 3 parallel microstrip lines coupled through a rotated slot are considered, and present results for  $|S_{41}|$  are compared

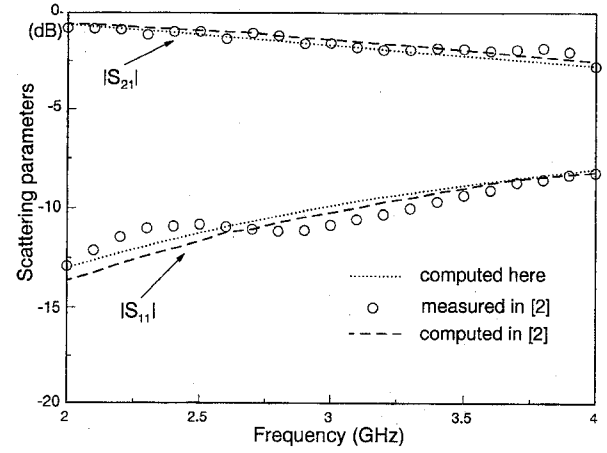


Fig. 2. Comparison of present computed results for  $|S_{11}|$  and  $|S_{21}|$  with measured and computed results reported by Herscovici and Pozar in [2] for the case of parallel microstrip lines with a perpendicular slot ( $\theta_l = \theta_u = 0^\circ$ ,  $\epsilon_{rl} = \epsilon_{ru} = 2.22$ ,  $h_l = h_u = 0.762$  mm,  $w_l = w_u = 2.54$  mm,  $l_s = 15$  mm, and  $w_s = 1.1$  mm).

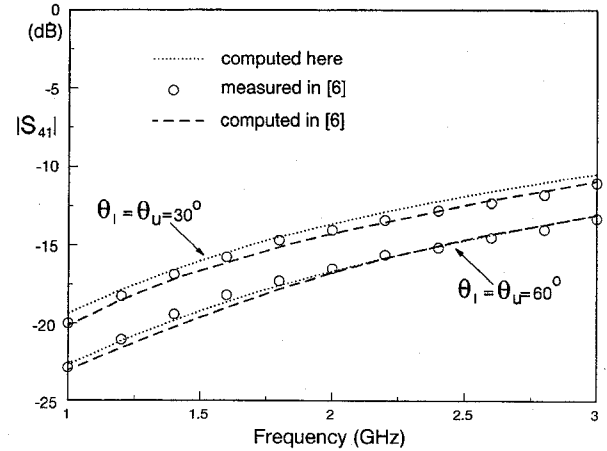


Fig. 3. Comparison of present computed results for  $|S_{41}|$  with measured and computed results reported by Ono *et al.* in [6] for the case of parallel microstrip lines coupled through a rotated slot ( $\theta_l = \theta_u$ ,  $\epsilon_{rl} = \epsilon_{ru} = 2.22$ ,  $h_l = h_u = 0.762$  mm,  $w_l = w_u = 2.54$  mm,  $l_s = 15$  mm, and  $w_s = 1.1$  mm).

with measured and computed data obtained by Ono *et al.* in [6]. Again, good agreement is obtained.

Now, effects of changing slot inclination angle on coupling between two coupled microstrip lines are investigated. As seen from Fig. 1, the upper microstrip line is oriented at an angle of  $\theta_u - \theta_l$  with respect to the lower microstrip line. Using the computer code developed based on the above analysis, for any fixed value of the line orientation angle  $\theta_u - \theta_l$ , scattering parameters are computed as a function of the slot inclination angle  $\theta_u$  relative to the upper microstrip line from  $-90$  to  $90^\circ$ . Computed results for the coupling  $|S_{31}|$  from port 1 to port 3 are shown in Fig. 4. It is apparent that coupling level changes over a wide range as  $\theta_u$  varies and, as expected, there is no coupling when the slot is parallel to either of microstrip lines. There exists a maximum coupling for a certain value of  $\theta_u = \theta_u^m$  and it decreases as the orientation angle between lines increases. The value of  $\theta_u^m$  increases as  $\theta_u - \theta_l$  increases,

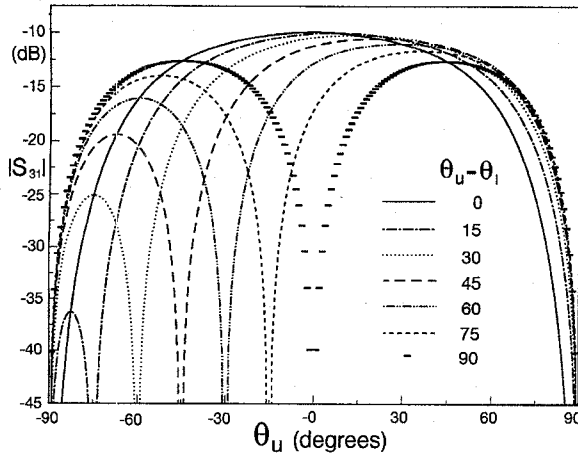


Fig. 4. Computed results for  $|S_{31}|$  as a function of  $\theta_u$  for different values of orientation angle between lines  $\theta_u - \theta_l$  ( $\epsilon_{rl} = \epsilon_{ru} = 10.2$ ,  $h_l = h_u = 0.635$  mm,  $w_l = w_u = 0.6$  mm,  $l_s = 3.5$  mm,  $w_s = 0.5$  mm,  $f = 10$  GHz).

and for the case where same substrates are used for both lines,  $\theta_u^m = 0.5(\theta_u - \theta_l)$ . For the parallel lines  $\theta_u - \theta_l = 0$ , there is only one peak in the coupling and the maximum coupling occurs when the slot is perpendicular to the lines  $\theta_u = 0^\circ$ . For the general case of  $\theta_u - \theta_l \neq 0$ , the other peak appears and the coupling at this peak is weaker than that at the first (main) peak, but it increases as  $\theta_u - \theta_l$  increases and reaches the same level as that in the main peak when two lines are perpendicular to each other  $\theta_u - \theta_l = 90^\circ$ .

In the actual design, the slot length is also a critical parameter for achieving the desirable coupling between two microstrip lines. Fig. 5 shows computed results for  $|S_{31}|$  and  $1 - |S_{11}|^2 - |S_{21}|^2 - |S_{31}|^2 - |S_{41}|^2$  as a function of slot length  $l_s$  up to the resonant slot length for three different frequencies. It is seen that the coupling can be controlled over a wide range by varying  $l_s$ . As  $l_s$  increases, the coupling  $|S_{31}|$  increases, but the loss  $1 - |S_{11}|^2 - |S_{21}|^2 - |S_{31}|^2 - |S_{41}|^2$  due to the radiation of the slot and surface waves excited in the substrates also increases. The loss increases more quickly when  $l_s$  is greater than  $\lambda_g/4$ . This radiation and surface wave loss can increase further when frequency increases, resulting in stronger interaction between components. Fortunately, if  $l_s$  is chosen to be less than  $\lambda_g/5$ , this loss is found to be negligible.

#### IV. CONCLUSION

In this letter we have developed the analysis of arbitrarily oriented slot-coupled microstrip lines by using the spectral

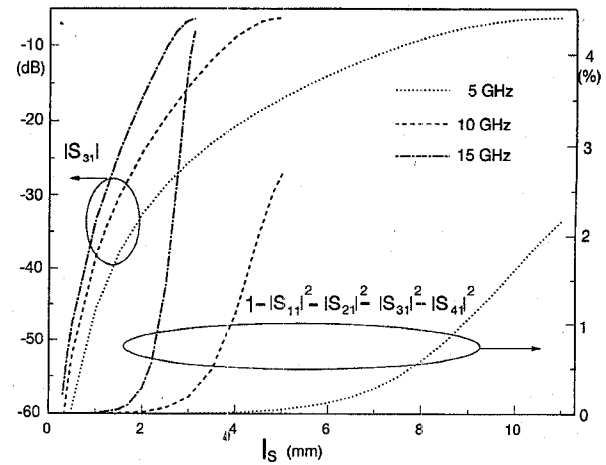


Fig. 5. Computed results for  $|S_{31}|$  and  $1 - |S_{11}|^2 - |S_{21}|^2 - |S_{31}|^2 - |S_{41}|^2$  as a function of slot length  $l_s$  up to the resonant slot length for three different frequencies ( $\theta_u = -\theta_l = 25^\circ$ ,  $\epsilon_{rl} = \epsilon_{ru} = 10.2$ ,  $h_l = h_u = 0.635$  mm,  $w_l = w_u = 0.6$  mm,  $w_s = 0.3$  mm).

domain approach and reciprocity theorem. The accuracy of the analysis has been verified by comparison with other available measured and computed data for the case of parallel microstrip lines. It has been shown that with suitable choice of the slot inclination angle the maximum coupling between two coupled lines can be achieved, but it decreases as the orientation angle between the lines increases. It has also been observed that coupling level can be controlled over a wide range by changing the slot inclination angle and slot length. With increasing slot length, the coupling increases, but the loss due to radiation and surface waves also increases.

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